Introduction to Representation Theory
Final Examination

April 29 2016

This exam is of 50 marks. Please read all the questions carefully and do not cheat. You are allowed to use *J-P Serre - Linear Representations of Finite Groups* and *Fulton and Harris - Representation Theory, A First Course.* Good luck! (50)

1. Let G be a group acting on a finite set X and ρ the corresponding permutation representation. Let χ_ρ be the corresponding character. For $g \in G$, show that (10)

$$\chi_\rho(g) = \mathrm{Card}(\{x \in X | g \cdot x = x\})$$

2. Compute the character table of $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. (10)

3. Let k be a finite field and $G = SL_2(k)$. Let H be the subgroup of upper triangular matrices in G,

$$H = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(k) \mid c = 0 \text{ and } ad = 1 \in k\}.$$

Let ω be a homomorphism $\omega : k^* \to \mathbb{C}^*$ and let χ_ω be the character of H given by

$$\chi_\omega(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}) = \omega(a)$$

Prove that $\mathrm{Ind}_H^G(\chi_\omega)$ is irreducible if and only if $\omega^2 \neq 1$. (10)

4. Show that the partition $3 = 2 + 1$ corresponds to the standard representation of S_3. (10)

5. Let $\lambda = 3 + 1$ and $\mu = 2 + 2$. Use Frobenius' formula to compute (10)

$$\chi_\lambda(C_\mu)$$

where C_μ is the conjugacy class corresponding to the partition $4 = 2 + 2$.